# Exam solutions 2017–12–13

ENM140, Game theory and rationality 2017

## Question 1

Consider a two-player simultaneous action game, where player I has actions A and B and player II has actions C and D. The payoffs are given by

| | C | D |
|---|---|---|
| A | (10, 10) | (8, 11) |
| B | (11, 8) | (9, 9) |

### 1.1

This is a coordination game.

**False**

### 1.2

This game has one mixed strategy Nash equilibrium and one pure strategy Nash equilibrium.

**False**

### 1.3

The pure strategy D is evolutionarily stable (in the strict sense).

**True**

Yes, because the game is symmetric and (B, D) is a Nash equilibrium.

### 1.4

The pure strategy profile where player I always plays B and player II always plays C is Pareto optimal.

**True**

### 1.5

The pure strategy profile where player I always plays B and player II always plays D is Pareto optimal.

**False**

### 1.6

This game can be represented in extensive form.

**True**

### 1.7

This is a game of imperfect information.

**True**

### 1.8

The game has infinitely many action profiles.

**False**

## Question 2

Consider a two-player simultaneous action game, where player I has actions A and B and player II has actions C and D. The payoffs are given by

| | C | D |
|---|---|---|
| A | $(3,\ 2)$ | $(0,\ 0)$ |
| B | $(0,\ 0)$ | $(2,\ 3)$ |

With this matrix as the single-round payoff matrix, consider the infinitely repeated game with average payoffs.

### 2.1

In the *single-round game*, there is a mixed strategy Nash equilibrium where players I and II, respectively, play the mixed strategies $(3/5,\ 2/5)$ and $(2/5,\ 3/5)$.

**True**

### 2.2

In the *infinitely repeated game*, there is a mixed strategy Nash equilibrium that is formed by the single-round mixed strategy profile (now played in every round) where players I and II, respectively, play the mixed strategies $(3/5,\ 2/5)$ and $(2/5,\ 3/5)$.

**True**

### 2.3

There is a strategy profile for the repeated game that has average payoff profile $(1,\ 1)$ and is a Nash equilibrium.

**False**

Not enforceable.

### 2.4

There is a strategy profile for the repeated game that has average payoff profile $(2,\ 2)$ and is a Nash equilibrium.

**True**

Use the Folk theorem.

### 2.5

There is a strategy profile for the repeated game that has average payoff profile $(1.5,\ 3)$ and is a Nash equilibrium.

**False**

Not feasible.

## Question 3

This question concerns the 3-round iterated Prisoner's dilemma with single-round payoff matrix as follows:

$$\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

For this problem we define a set of four strategies, $S = \{s_0, s_1, s_2, s_3\}$. The strategy $s_k$ is to conditionally cooperate for $k$ rounds, i.e., to cooperate the first $k$ rounds unless the opponent has defected in any earlier round.

Assume that both players choose their strategies from the set $S$ or as a mixed strategy based on strategies from the set $S$.

Then, the payoff profiles of the pure strategy profiles are as follows:

| | | Player 2 | | | |
|---|---|---|---|---|---|
| | | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
| Player 1 | $s_0$ | $(3,3)$ | $(7,2)$ | $(7,2)$ | $(7,2)$ |
| | $s_1$ | $(2,7)$ | $(5,5)$ | $(9,4)$ | $(9,4)$ |
| | $s_2$ | $(2,7)$ | $(4,9)$ | $(7,7)$ | $(11,6)$ |
| | $s_3$ | $(2,7)$ | $(4,9)$ | $(6,11)$ | $(9,9)$ |

### 3.1

The strategy $s_2$ is strictly dominated.

**False**

### 3.2

The strategy $s_3$ is strictly dominated.

**True**

Yes, e.g., by a suitable mixed strategy constructed from $s_2$ and $s_0$.

### 3.3

There is exactly one pure strategy Nash equilibrium and at least one mixed strategy Nash equilibrium.

**False**

### 3.4

The strategy profile where both players always play $s_0$ is a subgame perfect equilibrium.

**True**

### 3.5

The strategy profile where both players always play $s_1$ is a subgame perfect equilibrium.

**False**