Game theory and rationality (ENM140) exam 2017–11–20

The number of marked statements was on average 13.2/36 and the median was 14/36.

The mean score was 6.9 and the median score was 7.

18/28 (64%) received 6 points or more.

Table 1 – This table gives an overview of the statements. Note that the statements are ordered by the fraction of students ($n = 28$) who marked the statement as correct. Please take an extra look at the statements where less than half of the students marked the correct answer. Did you? Why/why not?

	Correct answer	Fraction marked	Fraction correct	Mean score	< half correct
1.1	1	18%	18%	0.18	*
4.6	1	18%	18%	0.18	*
7.5	1	21%	21%	0.21	*
6.5	1	25%	25%	0.25	*
4.1	1	39%	39%	0.39	*
1.2	0	61%	39%	-0.61	*
7.3	1	43%	43%	0.43	*
7.1	1	43%	43%	0.43	*
1.5	1	46%	46%	0.46	*
4.4	1	57%	57%	0.57	
3.2	0	43%	57%	-0.43	
5.1	0	36%	64%	-0.36	
2.1	1	64%	64%	0.64	
6.6	1	68%	68%	0.68	
1.4	1	71%	71%	0.71	
5.3	0	25%	75%	-0.25	
4.3	1	79%	79%	0.79	
3.6	1	79%	79%	0.79	
1.3	0	21%	79%	-0.21	
7.6	0	18%	82%	-0.18	
5.2	1	82%	82%	0.82	
3.4	0	18%	82%	-0.18	
6.2	1	82%	82%	0.82	
7.4	1	82%	82%	0.82	
2.2	0	14%	86%	-0.14	
3.1	0	14%	86%	-0.14	
6.4	0	14%	86%	-0.14	
3.3	0	11%	89%	-0.11	
6.1	0	11%	89%	-0.11	
4.2	0	11%	89%	-0.11	
7.7	1	89%	89%	0.89	
7.2	0	7%	93%	-0.07	
6.3	0	7%	93%	-0.07	
2.3	0	4%	96%	-0.04	
4.5	0	4%	96%	-0.04	
3.5	0	0%	100%	0.00	

Question 1

Consider a symmetric two-player game with the following payoff matrix:

$$\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$$

1. ☒ The game has exactly three Nash equilibria.

   Yes, two pure and one mixed.

2. ☐ The game has exactly two Nash equilibria.
3. ☐ There are exactly two Pareto optimal strategy profiles.

   Only the strategy profile ((1, 0), (1, 0)) is PO. It has payoff profile (3, 3).

4. ☒ There are exactly four action profiles in this game.

   Each player has two possible actions, so there are four combinations.

5. ☒ There are infinitely many mixed strategy profiles in this game.

   The possible strategy profiles are all pairs of probability distributions over the two actions. If the strategies are $(p, 1-p)$ and $(q, 1-q)$, then each pair $p, q : 0 \leq p \leq 1 \text{and} 0 \leq q \leq 1$ corresponds to a strategy profile.

Question 2

Consider the infinitely repeated game with average payoffs and single-round payoff matrix as in Question 1 above.

1. ☒ There is a strategy profile for the repeated game that has average payoff profile (2, 2) and is a Nash equilibrium.

   Use the Folk theorem.

2. ☐ There is a strategy profile for the repeated game that has average payoff profile (1/2, 1/2) and is a Nash equilibrium.

Not enforceable.

3. ☐ There is a strategy profile for the repeated game that has average payoff profile $(3,\ 2)$ and is a Nash equilibrium.

Not feasible.

Question 3

Consider a symmetric 2-player game with n actions. Assume that the choice of actions is simultaneous, i.e., the two players do not know the other player's action when choosing.

1. ☐ Each player has exactly n^2 possible strategies.
2. ☐ There must be at least one pure strategy Nash equilibrium.

Rock-paper-scissors is a counter-example.

3. ☐ There must be infinitely many mixed strategy Nash equilibria.
4. ☐ There must be at least one evolutionarily stable strategy (in the strict sense).
5. ☐ The number of Pareto optimal strategy profiles must be larger than the number of Nash equilibria.
6. ☒ The game can be written in extensive form.

All games can be represented in extensive and in strategic form.

Question 4

Consider a two-player game with the following payoff matrix:

		Player 2	
		C	D
Player 1	A	(30, 70)	(50, 50)
	B	(80, 20)	(−100, 200)

1. ☒ All strategy profiles that are Nash equilibria for this game are also Pareto optimal.

This is a constant-sum game, so *all* strategy profiles are Pareto optimal, regardless of what the Nash equilibria are.

2. ☐ This game has at least one pure strategy Nash equilibrium.

3. ☒ This game has at least one mixed strategy Nash equilibrium.

4. ☒ One Nash equilibrium is the strategy profile where Player 1 plays action A with probability $p = \frac{9}{10}$ and Player 2 plays action C with probability $q = \frac{3}{4}$.

5. ☐ One Nash equilibrium is the strategy profile where Player 1 plays action A with probability $p = \frac{9}{10}$ and Player 2 plays action C with probability $q = \frac{1}{2}$.

6. ☒ One Pareto optimal strategy profile is where Player 1 plays action A with probability $p = \frac{9}{10}$ and Player 2 plays action C with probability $q = \frac{1}{2}$.

See Q1: All strategy profiles are Pareto optimal.

Question 5

Consider a population playing the rock-paper-scissors game with payoff matrix as follows:

	R	P	S
R	$(0,\ 0)$	$(-1,\ 1)$	$(1, -1)$
P	$(1, -1)$	$(0,\ 0)$	$(-1,\ 1)$
S	$(-1,\ 1)$	$(1, -1)$	$(0,\ 0)$

One strategy M in this game is the mixed one with uniform probability $1/3$ over all three actions.

1. ☐ The mixed strategy M is evolutionarily stable (in the strict sense).

2. ☒ The strategy profile (M, M) is a Nash equilibrium.

3. ☐ There are two or more Nash equilibria in this game.

Question 6

		Player 2				
		E	F	G	H	I
Player 1	A	$(8,1)$	$(1,3)$	$(8,0)$	$(5,0)$	$(1,1)$
	B	$(2,1)$	$(0,2)$	$(2,1)$	$(3,3)$	$(3,0)$
	C	$(4,1)$	$(0,4)$	$(5,1)$	$(3,0)$	$(1,2)$
	D	$(8,1)$	$(0,1)$	$(8,0)$	$(4,2)$	$(0,3)$

1. ☐ This game has no pure strategy Nash equilibrium.
2. ☒ This game has exactly one pure strategy Nash equilibrium.
3. ☐ This game has multiple pure strategy Nash equilibria.
4. ☐ The pure strategy where Player 1 plays action D is strictly dominated.
5. ☒ The pure strategy where Player 2 plays action E is strictly dominated.

   Yes, for example, playing 50% F and 50% H is better than 100% E, no matter what Player 1 does.

6. ☒ The pure strategy where Player 2 plays action G is strictly dominated.

   Yes, even by the pure strategy playing only F.

Question 7

Consider the following sequential game between two players.

In the first round player one receives a gold coin that she can decide to keep, and then the game is finished with payoff profile $(1,\ 0)$.

She may instead choose to let the game continue by passing the coin to the second player, and in that process the payoff to consider doubles so that there are now two coins for the second player to take a decision on. He may decide to keep the coins, then finishing the game with payoff profile $(0,\ 2)$, or he may also decide to let the game continue, giving back the coins to the first player so that there are 4 coins to consider in the third round, etc.

The game is played a maximum of 10 rounds, and in the 10th round the second player has no choice but just keeps the 2^9 coins that he was given while the first player gets 0. Thus, the payoff profile in this case is $(0,\ 512)$.

Let S^* be a mixed strategy profile where both players always hand over the coins to the other player, except in their last choices, i.e., rounds 8 and 9, where both players randomly decide to keep the coins with probability 1/2.

1. ☒ All Nash equilibria are characterized by payoff profile $(1,\ 0)$.
2. ☐ There is at least one strategy profile which is a subgame perfect Nash equilibrium and has the following properties: Player one decides to keep the coins in the first round. Player two decides to pass the coins in the second round (if he gets the chance), but keeps the coins in the fourth round (if the chance is given).

No, everyone walking away with their offer in any circumstance is the only subgame perfect equilibrium.

3. ☒ The strategy profile S^* (described above) has payoff profile $(64,\ 192)$.

Yes, because it gives expected payoff $\frac{1}{2} \cdot \frac{1}{2} \cdot 256$ to player 1 and $\frac{1}{2} \cdot 128 + \frac{1}{2} \cdot \frac{1}{2} \cdot 512$ to player 2.

4. ☒ This game can be represented in extensive form.
5. ☒ This game can be represented in strategic form.
6. ☐ There is only one Pareto optimal pure strategy profile.

The pure strategy profile that always leads to payoff profile (0, 512) is Pareto optimal, and so is the pure strategy profile that always leads to payoff profile (256, 0).

7. ☒ This is a game of perfect information.