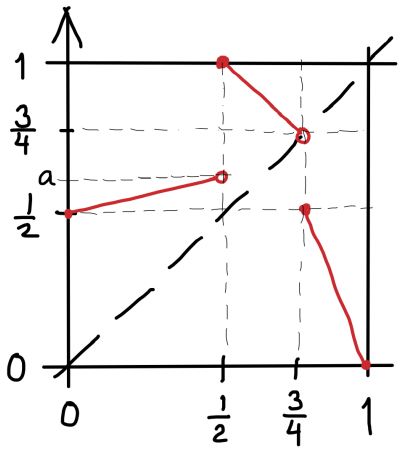
Homework 5

TIF150, Information theory for complex systems 2018

Let a piecewise linear map $f(x)$ be defined as illustrated in the figure below.



In mathematical terms, let $\frac{1}{2} \leq a \leq 1$, and

$$f(x) = \begin{cases} \frac{1}{2} + 2\left(a - \frac{1}{2}\right) x, & 0 \leq x < \frac{1}{2}, \\ \frac{3}{2} - x, & \frac{1}{2} \leq x < \frac{3}{4}, \\ \frac{1}{2} - 2\left(x - \frac{3}{4}\right), & \frac{3}{4} \leq x \leq 1. \end{cases}$$

Consider the dynamical system $x_{t+1} = f(x_t)$.

a) Start with $a = 1/2$ and discuss how the dynamics change when a is increased. Determine whether there is a stable fixed point, stable periodic orbit, or chaos. Is there a critical value for a, for which there is a change in dynamical characteristics?

b) Suppose now that $a = 1$. Determine the invariant measure that characterizes the chaotic behaviour, and calculate the Lyapunov exponent λ. Find a partition that is generating, and calculate the measure entropy from the symbolic dynamics. If you know that x_t is in the interval $[0, \frac{1}{2}]$ at time t, how much information do you gain if you learn that also x_{t+2} is in the same interval?

If you use equations or other results from the lectures or lecture notes, make sure to reference them and motivate why they may be used.

Hand in your solutions no later than Monday 5 March at 15:15. Late submissions will normally not be graded. You may hand in on paper in class, or by emailing a PDF named `yourcid.pdf`, e.g. `rasmuse.pdf`, to Rasmus: `rasmus.einarsson[at]chalmers.se`. Hand-written solutions are fine, but please take care to make them legible.