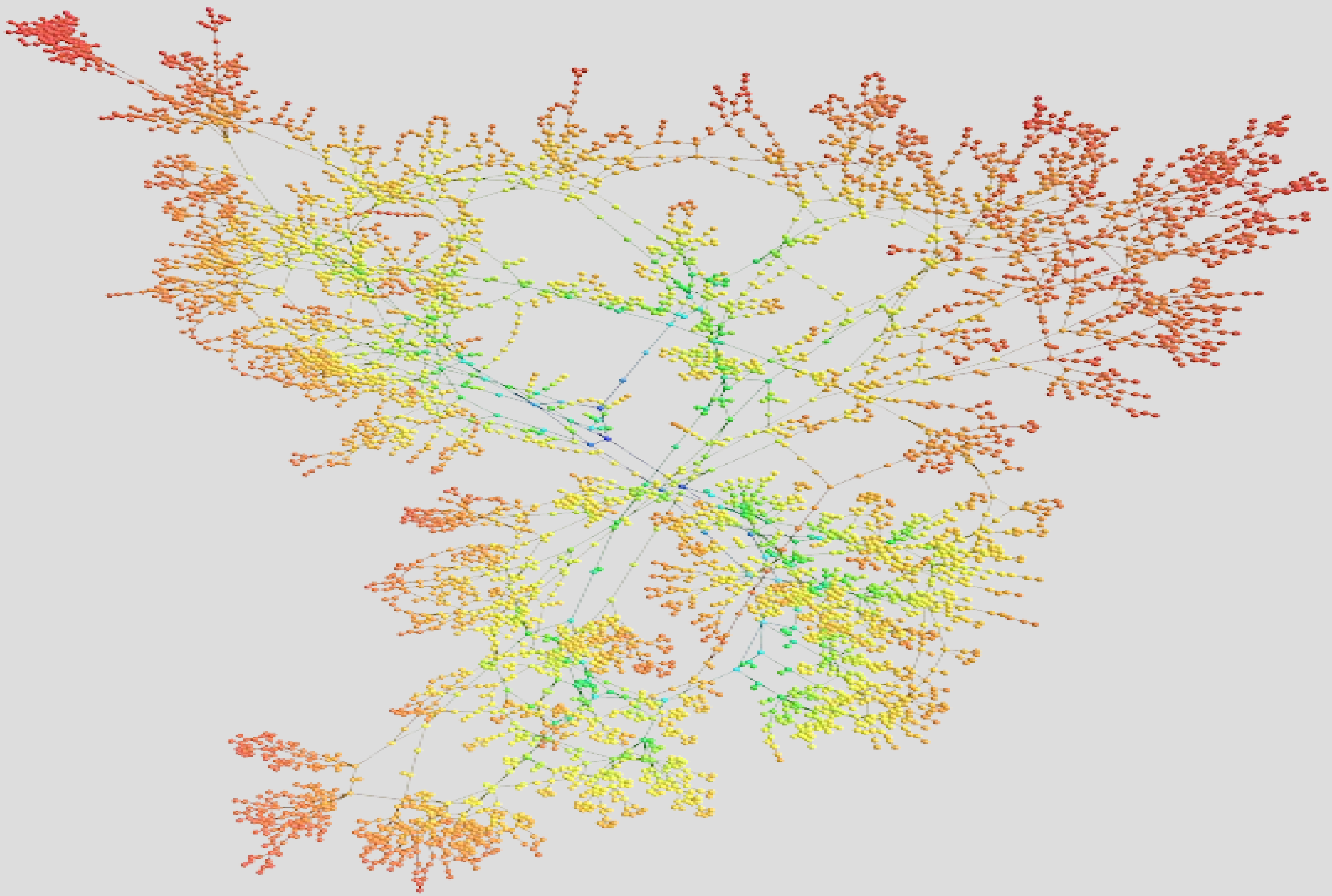
Immunizing well connected networks

Why random immunization does not work



Overview

- The SIS-modell
- Power-law distributed networks
- Random immunization
- Targeted immunization
- Conclusions

SIS-model
Susceptible-Infected-Susceptible

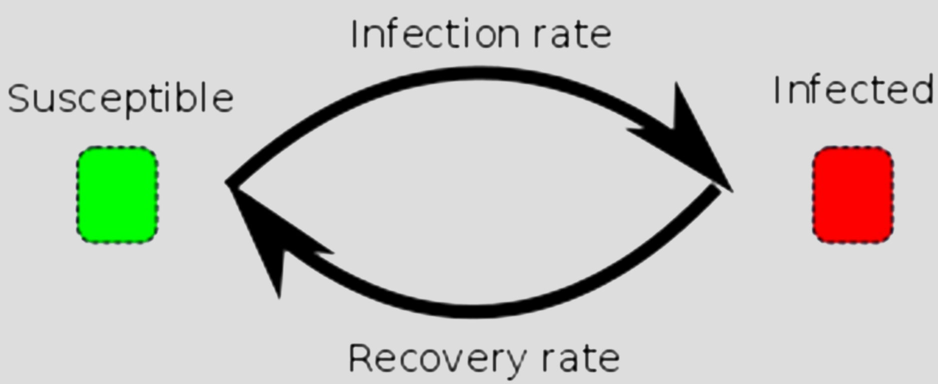
Infection rate = v
Recovery rate = δ = 1

Effective spreading rate = λ = v/δ = v

Critical epidemic threshold = λ_c

Homogenous networks:
$\lambda_c = \langle k \rangle^{-1}$

Power-law networks:
$\lambda_c = \langle k \rangle / \langle k^2 \rangle$



Power-law distribution

$P(k) \sim k^{-\gamma}$

- For $2<\gamma<3$ the risk of a node having a connectivity far above the mean $\langle k \rangle$ is significant.
- In fact, the fluctuations of connectivity, $\langle k^2 \rangle$, diverges, which means that $\lambda_c = \langle k \rangle / \langle k^2 \rangle = 0$, and an epidemy always occurs.

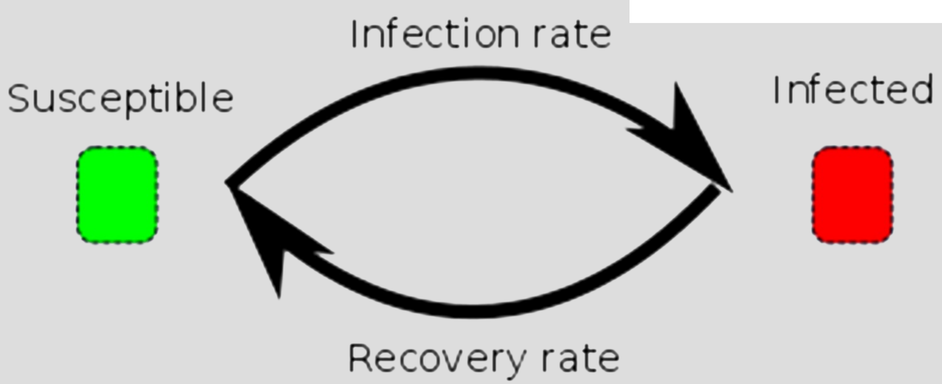
Homogeneous vs Heterogeneous

Homogeneous

$$\frac{d\rho(t)}{dt} = -\rho(t) + \lambda \langle k \rangle \rho(t) \left[1 - \rho(t)\right].$$

Heterogeneous

$$\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda k \left[1 - \rho_k(t)\right] \Theta(\rho(t)),$$



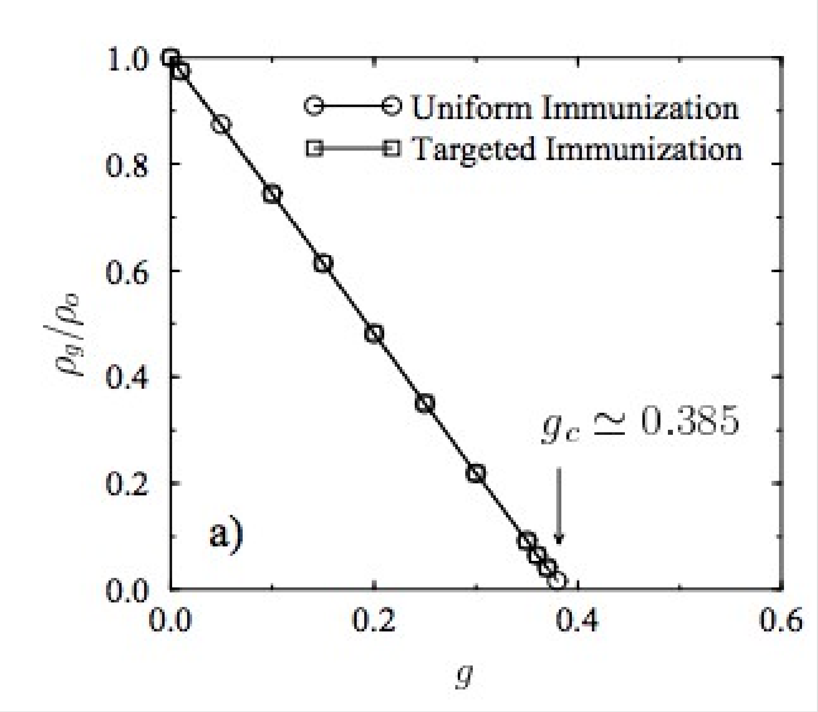
Random immunization

- Nodes are chosen randomly for immunization

$\lambda \rightarrow \lambda(1-g)$

g = immunization proportion

- Works for homogeneous networks, where a nodes conectivity can be approximated by the average connectivity <k>.



λ_c=0.25

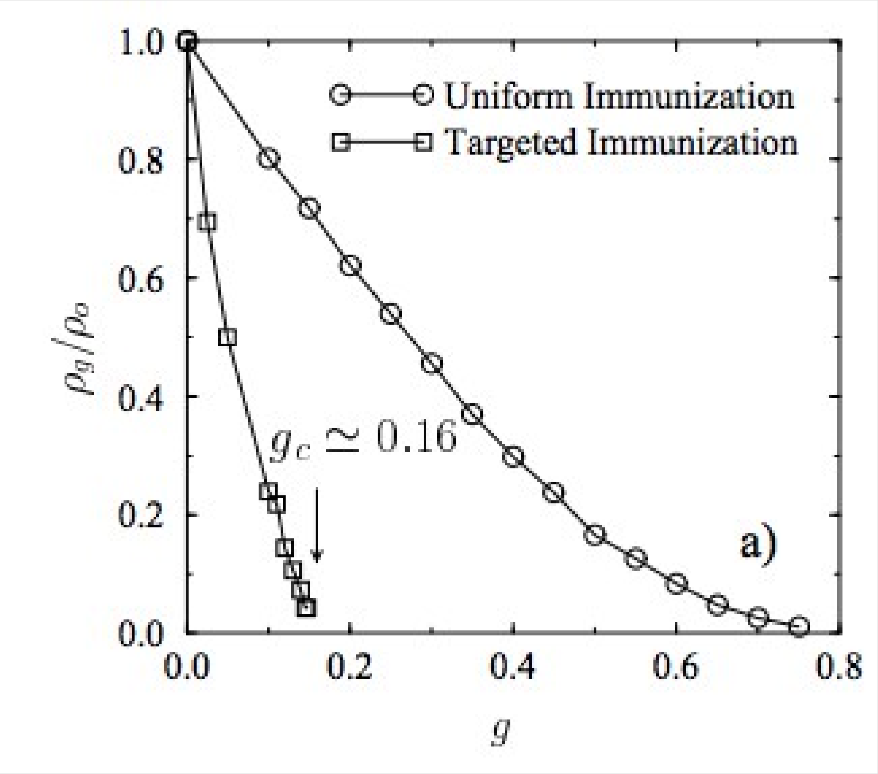
Random immunization

- Nodes are chosen randomly for immunization

$\lambda \rightarrow \lambda(1\text{-}g)$

g = immunization proportion

- Works for homogeneous networks, where a nodes conectivity can be approximated by the average connectivity <k>.



λ_c=0.25

Targetted immunization
and other methods

- Random immunization does not work for heterogeneous networks

Proportional immunization

- $g \rightarrow g_k$, $\lambda k \longrightarrow \lambda k(1\text{-}g_k)$
- $$\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda k \left[1 - \rho_k(t)\right] \Theta(\rho(t)),$$
- Set $\lambda k(1\text{-}g_k)$=const
- $g_k \sim 1\text{-}1/\lambda k$,
- Immunize groups with higher degree more

Targetted immunization

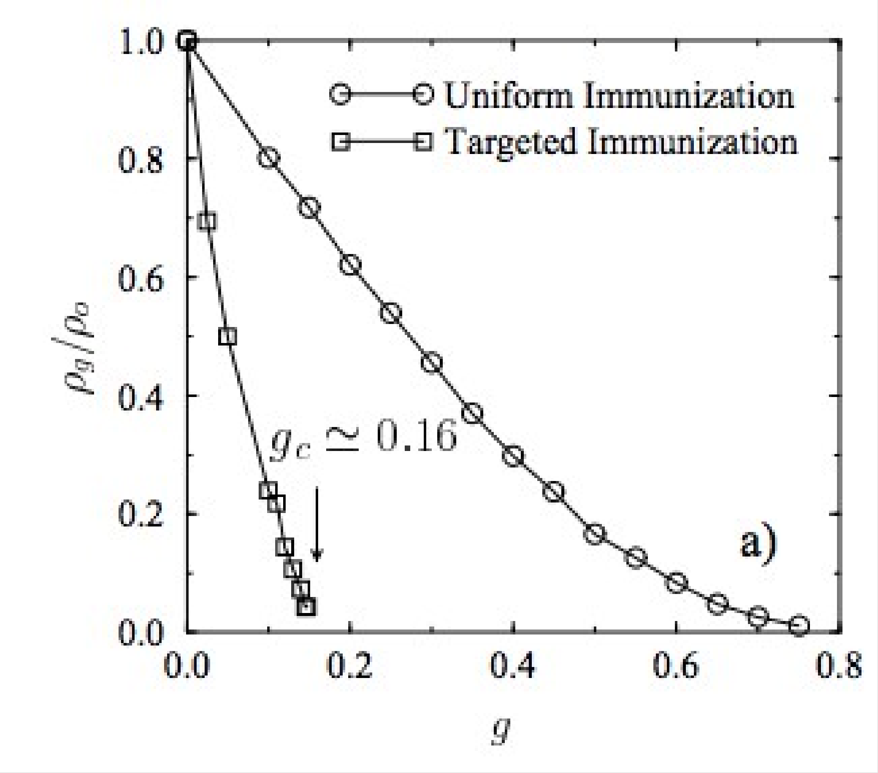
and other methods

Targetted immunization

- Even more effective method exists for power-law distributions
- g = fraction of most connected that get immunized

$$g = \sum_{k>k_t} P(k),$$

- p(g), probability that a link goes to an immunized node.



$$\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda k \left[1 - \rho_k(t)\right] \Theta(\rho(t)),$$

Conclusions

To successfully immunize a network, the high connectivity points must be found.

Can be hard for the internet, but very applicable in sexualy transmitted diseases