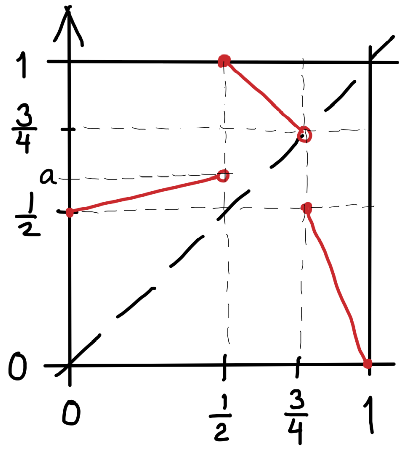
Homework 5

TIF150, Information theory for complex systems

Let a piecewise linear map $f(x)$ be defined as illustrated in the figure below.



In mathematical terms, let $\frac{1}{2} \leq a \leq 1$, and

$$f(x) = \begin{cases} \frac{1}{2} + 2\left(a - \frac{1}{2}\right)x, & 0 \leq x < \frac{1}{2}, \\ \frac{3}{2} - x, & \frac{1}{2} \leq x < \frac{3}{4}, \\ \frac{1}{2} - 2\left(x - \frac{3}{4}\right), & \frac{3}{4} \leq x \leq 1. \end{cases}$$

Consider the dynamical system $x_{t+1} = f(x_t)$.

a) Start with $a = 1/2$ and discuss how the dynamics change when a is increased. Determine whether there is a stable fixed point, stable periodic orbit, or chaos. Is there a critical value for a, for which there is a change in dynamical characteristics?

b) Suppose now that $a = 1$. Determine the invariant measure that characterizes the chaotic behaviour, and calculate the Lyapunov exponent λ. Find a partition that is generating, and calculate the measure entropy from the symbolic dynamics. If you know that x_t is in the interval $[0, \frac{1}{2}]$ at time t, how much information do you gain if you learn that also x_{t+2} is in the same interval?

If you use equations or other results from the lectures or lecture notes, make sure to reference them and motivate why they may be used.